

The spin jumping in the context of a QCD effective model

G. B. de Gracia

^{1,2}*Instituto de Física Teórica, UNESP - São Paulo State University,
P. O. Box 70532-2, 01156-970, São Paulo, SP, Brazil.
gb9950@gmail.com*

Abstract

The tensor formulation for the effective theory of QCD vector resonances, whose model we denote by TEVR, is given by an antisymmetric tensor field and describes spin 1 particles. Our goal is to show, by different approaches, that the Abelian version of this model presents the so called “spin jumping” when we consider its massless limit. Classically we find, by the use of the equations of motion and the Hamiltonian constraint analysis, that the massive phase of the model describes spin 1 particles while its massless phase describes spin 0 particles. By the quantum point of view we derive these conclusions via tree level unitarity analysis and the master action approach.

Keywords: Degrees of freedom, spin jumping, discontinuity.

PACS number:

1 Introduction

According to Ref. 1 and 2, we know that models described by antisymmetric fields present a discontinuity in its degrees of freedom, which we call spin jumping, when considering the massless limit. This is due to the fact that in the massless phase p forms are dual to $D - p - 2$ forms while in the massive phase the p forms are dual to $D - p - 1$ forms. There are also another cases where this discontinuity may occur, for example, near the massless limit of the topologically massive spin 1 model of Ref. 3 in which a four dimensional BF term, which do not have any local degrees of freedom Ref. 4, becomes dominant. Alternatively, it may happen in some models that the massless equations of motion and the boundary conditions determines just a trivial solution to the fields while its massive phase possesses a definite spin content, as we can see in the appendix A.

The main goal of this article is to verify the occurrence of this phenomena in the context of the Abelian version of the TEVR model, which is an effective theory for the vector resonances of particular importance in QCD as we can see in Refs. 5, 6 and 7. More specifically, it describes the resonances 1^{--} which are excited states of quark bound states with spin 1. So, for some circumstances it is easier to study these bound states as pointwise particles via some effective action. We may mention the fact that due to the spin discontinuity of the model some approximations must be done carefully in order to have a consistent effective description. For example, in performing some calculations in the S matrix for high momentum one often neglect the particle masses, but in the present context this approximation cannot be so radical since in this limit the TEVR model predict a different spin behaviour to the effective pointwise particle which is related, among other facts, to a change in its interparticle potential. In fact, this potential presents a DVZ discontinuity as we can see in Ref.8.

The model is described by an antisymmetric field $B_{\mu\nu}(x)$ and its massive phase describes spin 1 particles, while its massless phase describes spin 0 excitations. An important motivation to this article is that a complete knowledge of the spin jumping phenomena may give us some insight to use this degree of freedom discontinuity in the context of an effective description for phase transitions controlled by the mass parameter of the TEVR model. It is also interesting to mention, as it was expected due the previous discussion, that the Kalb Ramond model, see Refs. 9 and 10, which also uses an antisymmetric field, presents the same kind of degree of freedom discontinuity studied in Ref. 1. We explore this fact to do some general observations and analogies throughout this article.

In order to analyze the spin jumping phenomena for the TEVR model we use different approaches: Regarding the Sec. 2, the “on shell” degrees of freedom of the model, in its massive and massless phases, are inferred by the constraints of the equations of motion. In Sec. 3, we present a counting of degrees of freedom for the massive/ massless phases of the model which is based on the Hamiltonian formalism, more

specifically, the Dirac-Bergmann algorithm. The Sec. 4 is devoted to the inference of the ‘spin jumping’ by the quantum point of view, through the tree level unitarity analysis. In Sec. 5, the master action approach is used as a way to show the dual relation between the TEVR and the Maxwell-Proca model while its massless phase can be related to a real scalar field model. This can be understood as an another method to infer the spin jumping. Finally, in Sec. 6 we conclude. Technical details are relegated to the appendices.

Natural units are used throughout and the Minkowski metric is $diag(-1, +1, \dots, +1)$.

2 Spin Jumping and the Equations of Motion

In this section we show that the equations of motion of the TEVR model can be used as a way to infer the ‘on-shell’ degrees of freedom propagated by this model in its massive and massless phases.

2.1 The massive phase

The massive phase of the TEVR model is given by the action below:

$$S = \int d^D x \left[\partial^\mu B_{\mu\nu} \partial_\beta B^{\beta\nu} + \frac{m^2}{2} B_{\mu\nu} B^{\mu\nu} \right] \quad (1)$$

Where $B_{\mu\nu} = -B_{\nu\mu}$.

The equations of motion can be obtained by the variational principle and can be rewritten in terms of the antisymmetric spin operators ¹:

$$\left[(m^2 - \square) P_{\mu\nu\alpha\beta}^{1e} + m^2 P_{\mu\nu\alpha\beta}^{1b} \right] B^{\mu\nu}(x) = 0 \quad (2)$$

By acting with P^{1b} in the above equation and using the orthogonality of the antisymmetric spin operators we can show:

$$m^2 P_{\mu\nu\alpha\beta}^{1b} B^{\mu\nu}(x) = 0 \quad (3)$$

The constraint above removes $\frac{(D-1)(D-2)}{2}$ degrees of freedom from the $B_{\mu\nu}$ field. This result allows us to conclude that an antisymmetric field that obeys this constraint propagates $D - 1$ degrees of freedom, which is characteristic of a massive spin 1 particle in D dimensions.

Now, we need to show that the $B_{\mu\nu}(x)$ field components obeys the Klein-Gordon equation. It can be done by using the identity² $P_{\mu\nu}^{(1e)\alpha\beta} = I_{\mu\nu}^{\alpha\beta} - P_{\mu\nu}^{(1b)\alpha\beta}$ and the equation (3) applied in the equation (2).

2.2 The massless phase

The massless phase of the TEVR model presents a gauge symmetry in analogy to the massless Kalb Ramond theory. We could, at first, fix this symmetry to obtain the correct number of its degrees of freedom. However, this is not the easiest way to do it. So, we follow a more direct approach which is given below.

The equations of motion for the massless limit of the TEVR model can be written as:

$$\square P_{\mu\nu\alpha\beta}^{1e} B^{\mu\nu}(x) = 0 \quad (4)$$

The fact that this model, as its equations of motion, can be written in terms of just one spin operator leads us to conclude that it must have a gauge symmetry of the form ³:

$$\delta B_{\mu\nu}(x) = \partial^\alpha \Lambda_{\alpha\mu\nu}(x) \quad ; \quad \Lambda_{\alpha\mu\nu}(x) = \Lambda_{[\alpha\mu\nu]}(x) \quad (5)$$

In order to immediately infer the degrees of freedom propagated by the massless $B_{\mu\nu}$ field, we put aside the gauge symmetry discussion and focus on the informations contained in the equations of motion.

¹Which are defined in the appendix.

²The symbol $I_{\mu\nu\alpha\beta}$ refers to the rank 4 antisymmetrized identity whose form lies in the appendix.

³Square brackets denote antissymmetrization.

So, we use the fact that they can be rewritten in the following manner:

$$\partial^\mu(\partial_\gamma B^{\gamma\nu}) - \partial^\nu(\partial_\gamma B^{\gamma\mu}) = 0 \quad (6)$$

From the above equations we can see that the divergence of the $B_{\mu\nu}$ field is the gradient of a scalar field $\partial_\gamma B^{\gamma\nu} = \partial^\nu \phi$. By contracting $B_{\mu\nu}$ with two derivatives and from its antisymmetry we can conclude:

$$\partial_\nu \partial_\gamma B^{\gamma\nu}(x) = \partial_\nu \partial^\nu \phi(x) \rightarrow \square \phi(x) = 0 \quad (7)$$

Therefore, the equations of motion of the massless TEVR model are equivalent to a harmonic scalar field in analogy to what happens to the massless Kalb Ramond model in $D = 3 + 1$ dimensions according to Ref. 11. Thus, we show, by the use of the equations of motion, that the spin jumping phenomena occurs, at least on the mass shell of the TEVR model. It is important to mention that the degree of freedom analogy between the TEVR and the Kalb Ramond model holds just in $D = 3 + 1$ dimensions⁴.

3 Hamiltonian Analysis in D Dimensions

The Hamiltonian analysis, in particular the Dirac-Bergmann algorithm, see Refs. 12 and 13, provides us with a powerful method to obtain the degrees of freedom of some theory. To proceed with this analysis we divide it again in two parts, for the massive and for the massless phases of the theory.

3.1 Hamiltonian analysis of the massive phase

The Lagrangian density of the TEVR model is given below:

$$\mathcal{L} = (\partial^\mu B_{\mu\nu})^2 + \frac{m^2}{2} B_{\mu\nu} B^{\mu\nu} \quad (8)$$

Where we use the standard notation $X_\mu X^\mu \equiv (X^\mu)^2$.

The canonical momenta follows from the expression⁵ $\pi^{\mu\nu} = \partial \mathcal{L} / \partial \dot{B}_{\mu\nu}$. So, we have:

$$\pi^{ij} = 0 \quad ; \quad \pi^{0i} = 2(\dot{B}_{0i} + \partial^j B_{ji}) \quad (9)$$

From the definition of the canonical momenta we can obtain the primary constraints:

$$\alpha^{ij} \equiv \pi^{ij} \approx 0 \quad (10)$$

The canonical Hamiltonian is given by the expression:

$$\mathcal{H}_c = \pi^{0i} \dot{B}_{0i} - \mathcal{L} = \frac{(\pi^{0i})^2}{4} - \pi^{0i} \partial^j B_{ji} + (\partial^i B_{0i})^2 - \frac{m^2}{2} (B_{ij})^2 + m^2 (B_{0j})^2 \quad (11)$$

To obtain the primary Hamiltonian we add Lagrange multipliers to each of the primary constraints:

$$H_p = \int d^{D-1}x \left[\mathcal{H} + \lambda_{ij} \alpha^{ij} \right] \quad (12)$$

⁴This is due to the fact that the dynamical part of Kalb Ramond model is found in the space projected by the operator $P_{\mu\nu\alpha\beta}^{1b}$. So its massive phase has $\frac{(D-1)(D-2)}{2}$ degrees of freedom, which is equivalent to spin 1 just in $D = 3 + 1$ dimensions. Its massless phase can be shown to be dual to a scalar field just in $D = 3 + 1$ dimensions see Ref. 11.

⁵Where $\dot{B}_{\mu\nu}$ means $\partial_0 B_{\mu\nu}$.

To proceed with the Hamiltonian analysis it is necessary to evaluate the consistency of the constraints but to do so we first need to know the fundamental non-vanishing Poisson brackets of the theory:

$$\{B_{\mu\nu}(x), \pi^{\alpha\beta}(y)\} = \delta^{(D-1)}(x-y) \frac{(\delta_\mu^\alpha \delta_\nu^\beta - \delta_\mu^\beta \delta_\nu^\alpha)}{2} \quad (13)$$

Now we may check the vanishing of the constraint's time evolution:

$$\dot{\alpha}^{ij}(x) = \{\alpha^{ij}(x), H_c\} = \left[\frac{F^{ij}(\pi^{0i})}{2} - m^2 B^{ij} \right] = 0 \quad (14)$$

Where we have that $F^{ij}(\pi^{0i}) \equiv \partial^i \pi^{0i} - \partial^j \pi^{0j}$.

In order to guarantee the consistency of the primary constraints we are lead to consider the secondary ones:

$$\Phi^{ij}(x) \equiv \frac{F^{ij}(\pi^{0i})(x)}{2} - m^2 B^{ij}(x) \approx 0 \quad (15)$$

The time evolution of the new constraints above are given by:

$$\dot{\Phi}^{ij}(x) = \{\Phi^{ij}(x), H_c\} = 2m^2 F_{ij}(B^{0i}) - m^2 \lambda^{ij} = 0 \quad (16)$$

We can conclude from the above result that the Dirac-Bergmann algorithm came to its end since the Lagrange multipliers are determined. It is not difficult to check that all the constraints are of second class which is related to the fact that the massive TEVR do not have gauge symmetry (The Dirac brackets of the system are given in the appendix C).

Regarding the counting of degrees of freedom, we had initially $D(D-1)$ phase space degrees of freedom, namely the fields and its canonical momenta. When we take the constraints into account we have that the primary ones $\alpha^{ij}(x)$ remove $\frac{(D-1)(D-2)}{2}$ degrees of freedom, while the secondary ones remove this same quantity. Thus, in the end of the day we have $2(D-1)$ phase space degrees of freedom which are compatible to a spin 1 particle in D dimensions in according to the previous section⁶.

Considering the constraints as strong equalities the Hamiltonian become positive definite which guarantees the classical stability of the model:

$$H_c = \int d^3x \left[\frac{(\pi^{0i})^2}{4} + (\partial^i B_{0i})^2 + \frac{m^2}{2} (B_{ij})^2 + m^2 (B_{0j})^2 \right] \geq 0 \quad (17)$$

3.2 Hamiltonian analysis of the massless phase

The Lagrangian density of the massless phase of the TEVR model is given by:

$$\mathcal{L} = (\partial^\mu B_{\mu\nu})^2 \quad (18)$$

Since the kinetic part of the massless phase of the model is the same that of the massive phase their primary constraints must be the same. The Hamiltonian density is given by the following expression:

$$\mathcal{H}_c = \frac{(\pi^{0i})^2}{4} - \pi^{0i} \partial^j B_{ji} + (\partial^i B_{0i})^2 \quad (19)$$

The primary Hamiltonian is obtained by associating Lagrange multipliers to each of the constraints:

⁶We must remember that the phase space has double the dimensions of the configuration space.

$$H_p = \int d^{D-1}x \left[\mathcal{H}_c + \lambda_{ij} \alpha^{ij} \right] \quad (20)$$

The first difference between the constraint structure of the massive and the massless versions of the TEVR models lies on its primary constraints consistency conditions:

$$\dot{\alpha}^{ij}(x) = \{\alpha^{ij}(x), H_c\} = F^{ij}(\pi^{0i}) = 0 \quad (21)$$

From the above expression we conclude that secondary constraints are needed:

$$\Omega^{ij} \equiv F^{ij}(\pi^{0i}) \approx 0 \quad (22)$$

The analysis of those new constraints is more intricate than the previous ones. The reason is due to the fact that we are lead to the issue of reducibility, see Ref. 21, due to the existence of the relations ⁷:

$$P_{ijmn}^{1b} \Omega^{mn} = 0 \quad (23)$$

The presence of reducibility is related to the fact that to quantize the system, one must introduce extra ghost terms in the gauge fixed Lagrangian in order to have a system without any local freedom.

We could, at principle, perform a Hamiltonian analysis inside the context of reducibility but we find it simpler to follow the alternative way of deriving a general solution to the constraints (22). So, in the constraint surface we have:

$$\pi^{0i}(x) = \partial^i \gamma(x) \quad (24)$$

Where $\gamma(x)$ is a scalar field with an appropriate mass dimension.

The Ω^{ij} constraint forces π^{0i} to be purely longitudinal. So it removes $D - 2$ degrees of freedom from the model, a diferent quantity that is removed by the secondary constraints in the massive phase.

The consistency conditions are identically satisfied:

$$\dot{\Omega}^{ij}(x) = \{\Omega^{ij}(x), H_c\} = 0 \quad (25)$$

Thus, the algorithm comes to its end and we can infer that there is the presence of gauge symmetry since the constraints are now of the first class and the Lagrange multipliers are indeterminated which means that the theory has some arbitrary local freedom.

The degree of freedom count is done by removing from the initial $D(D - 1)$ dimensional phase space two degrees of freedom for each first class constraint. The necessity for removing the double of degrees of freedom that would be naively removed by the constraints is due to the fact that for each first class constraint we should fix one corresponding gauge fixing condition to the field $B_{\mu\nu}(x)$ in order to have a uniquely determined dynamics for it.

Regarding the counting of the degrees of freedom we have in the end of the day two phase space degrees of freedom, which corresponds to just one in the configuration space, so the massless limit of the TEVR model describes a spin 0 particle.

The classical stability of the theory is guaranteed by the fact that in the constraint surface the Hamiltonian is positive definite:

$$\mathcal{H}_c = \frac{(\pi^{0i})^2}{4} + (\partial^i B_{0i})^2 \geq 0 \quad (26)$$

From the Hamiltonian analysis we can recover the results obtained by the equations of motion through a more rigorous way. Our next step is to show that at the quantum level we can also infer this degree of freedom discontinuity when considering the massless limit.

⁷ P_{ijmn}^{1b} is the spatial part of the antisymmetric spin projector. See appendix B

4 Tree Level Unitarity Analysis

In this section we show that by means of the unitarity analysis, see Refs. 14 and 15, we can infer the physical degrees of freedom of the TEVR model by identifying the form of the terms that contributes to the residue of the saturated amplitude. First we will treat the massive and then its massless phase to show the occurrence of the spin jumping under the quantum point of view. The procedure of verifying the tree level unitarity consists in coupling an external classical source to the fields of the free Lagrangian. This interaction term will generate contributions to the functional generator which can be expressed by means of tree diagrams with this source attached to the external lines. Since this source is weakly coupled to the theory, in order to verify its unitarity it is enough to look at the positivity of the imaginary part of its first non trivial contribution. In practice it can be done by using a simple application of the Cutkosky rules.

4.1 On the unitarity of the massive phase

In order to perform the unitarity analysis of TEVR model it is useful to express its action in terms of the antisymmetric spin operators and also add a source term:

$$S = \int d^D x \left[(\partial^\mu B_{\mu\nu})^2 + \frac{m^2}{2} B_{\mu\nu} B^{\mu\nu} + B_{\mu\nu} T^{\mu\nu} \right] = \int d^D x \left\{ \frac{B^{\mu\nu}}{2} \left[(m^2 - \square) P_{\mu\nu\alpha\beta}^{1e} + m^2 P_{\mu\nu\alpha\beta}^{1b} \right] B^{\alpha\beta} + B_{\mu\nu} T^{\mu\nu} \right\} \quad (27)$$

Where $T_{\mu\nu} = -T_{\nu\mu}$.

The action differential operator, expressed in terms of the spin operators, reads:

$$\hat{O}_{\mu\nu\alpha\beta} = \frac{[(m^2 - \square) P_{\mu\nu\alpha\beta}^{1e} + m^2 P_{\mu\nu\alpha\beta}^{1b}]}{2} \quad (28)$$

The saturated amplitude can be obtained by contracting the inverse of this differential operator with the antisymmetric sources. In the momentum space this calculation becomes straightforward. So, the amplitude is given by:

$$\mathcal{A}_{(m \neq 0)}(k) = -\frac{i}{2} T^{\mu\nu*}(k) \hat{O}_{\mu\nu\alpha\beta}^{-1}(k) T^{\alpha\beta}(k) = -iT^{\mu\nu*}(k) \left[\frac{P_{\mu\nu\alpha\beta}^{1e}}{(m^2 + k^2)} + \frac{P_{\mu\nu\alpha\beta}^{1b}}{m^2} \right] T^{\alpha\beta}(k) \quad (29)$$

$$\mathcal{A}_{(m \neq 0)}(k) = -i \left[\frac{-|k^\mu T_{\mu\nu}|^2}{m^2(k^2 + m^2)} + \frac{|T_{\mu\nu}|^2}{m^2} \right] \quad (30)$$

The unitarity analysis is done by verifying the signal of the imaginary part of the residue of the saturated amplitude. From the above expression we can conclude that the term that contributes to the residue comes from the contribution of the operator $P_{\mu\nu\alpha\beta}^{1e}$. This operator has the property of projecting in a $D - 1$ dimensional space:

$$\Im \lim_{k^2 \rightarrow -m^2} (k^2 + m^2) \mathcal{A}_{(m \neq 0)}(k) = \frac{|k^\mu T_{\mu\nu}|^2}{m^2} \quad (31)$$

In order to show that the above quantity is positive definite, which means that the theory is unitary, we use the definition:

$$|k^\mu T_{\mu\nu}(k)| \equiv J_\nu^T(k) \quad (32)$$

Where the index “ T ” designates transversality.

A convenient frame to perform the calculation of (31) is given below:

$$k_\mu = (m, 0, 0, \dots, 0) \quad ; \quad k^\mu J_\mu^T(k) = 0 \quad ; \quad J_0^T = 0 \quad (33)$$

In this frame we can use the transversality of $J_\mu^T(k)$ to show that $J_0^T = 0$. Thus, the expression of (31) becomes positive definite and we can infer that the model is tree level unitary:

$$\Im \lim_{k^2 \rightarrow -m^2} (k^2 + m^2) \mathcal{A}_{(m \neq 0)}(k) > 0 \quad (34)$$

From this result we can conclude that the contribution to the unitarity comes from the P^{1e} spin operator which projects the tensor field in its spin 1 sector. This result is in accordance to our earlier results which were obtained by classical methods.

4.2 On the unitarity of the Massless phase

The unitarity analysis of the massless phase of the TEVR model is more intricate since its differential operator is not invertible due to its gauge freedom:

$$\delta B_{\mu\nu} = \partial^\alpha \Lambda_{[\alpha\mu\nu]} \quad (35)$$

Thus, in order to proceed with such an analysis it is necessary to add a gauge fixing term which will allow us to invert the differential operator. The gauge condition we choose is:

$$G_{\lambda\mu\nu} = \partial_\lambda B_{\mu\nu} + \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} = 0 \quad (36)$$

The fact that the system has gauge symmetry is related to constraints in the sources. Those constraints can be obtained by requiring that the source term of the model must be invariant by the same symmetry transformations that leaves its quadratic part invariant:

$$\int \delta \mathcal{L}_{source} d^D x = \int \delta B_{\mu\nu} T^{\mu\nu} d^D x = 0 \rightarrow T_{\mu\nu}(k) = k_\mu J_\nu - k_\nu J_\mu \quad (37)$$

The gauge fixing term which is totally projected on the P^{1b} operator has the form:

$$\mathcal{L}_{g.f} = \frac{\lambda}{2} G_{\lambda\mu\nu}(B) G^{\lambda\mu\nu}(B) \quad (38)$$

The gauge fixed Lagrangian reads:

$$\mathcal{L} = (\partial^\mu B_{\mu\nu})^2 + \frac{\lambda}{2} G_{\lambda\mu\nu}(B) G^{\lambda\mu\nu}(B) \quad (39)$$

By using the form (37) for the sources and the same procedure adopted in the previous subsection we obtain the saturated amplitude:

$$\mathcal{A}_{(m=0)}(k) = -i \frac{T^{\mu\nu*}(k) P_{\mu\nu\alpha\beta}^{1e} T^{\alpha\beta}(k)}{k^2} = -i \frac{(k^4 |j_\beta|^2 - k^2 |k^\alpha J_\alpha|^2)}{k^4} \quad (40)$$

From the above expression we have that:

$$\Im \lim_{k^2 \rightarrow 0} k^2 \mathcal{A}_{(m=0)}(k) > 0 \quad (41)$$

This result allow us to conclude that the massless phase of the TEVR model is unitary at tree level . Regarding the degrees of freedom we note that the contribution to the residue of the saturated amplitude comes from a term of the form $|k^\alpha J_\alpha|^2$ which can be written as $k^2 j^{\nu*} \omega_{\nu\mu} J^\mu$ with $\omega_{\nu\mu}$ being the vector spin 0 operator (see appendix B.).

Therefore, we note that the massless phase contribution to unitarity comes from a term that can be expressed by the vector spin 0 operator. This result is in accordance with the analysis of the previous sections and when compared to the Ref. 10 it can be understood as a quantum level spin jumping.

5 Master Action and Degrees of Freedom

The goal of this section is to obtain a master action, see Refs. 16 and 17, that relates the TEVR with the Maxwell-Proca above in D dimensions and one that relates its massless limit with the action of a scalar field.

The relation between the TEVR and the Maxwell-Proca model is already inferred, but under different approaches as can be seen in Ref. 5. Our goal is to show that this relation can be also obtained by using a master action. From these results we can finally compare them to the ones obtained for its massless phase and find, now by the master action approach, the already mentioned degree of freedom discontinuity.

By adding sources to the master action fields it is possible, for example, to compare its two point functions and obtain a dual map between them. In the massless phase we show that there is a problem in the determination of this dual map, a fact that has an direct analogue when considering the massless Kalb Ramond model.

5.1 Massive phase and the Maxwell-Proca model

The master action that interpolates the TEVR and the Maxwell-Proca model has the form:

$$S_M^{(m \neq 0)} = \int d^D x \left[m^2 B_{\mu\nu} B^{\mu\nu} + 2m(\partial^\mu B_{\mu\nu})A^\nu - \frac{m^2}{2} A^\mu A_\mu + A_\nu J^\nu + B_{\mu\nu} T^{\mu\nu} \right] \quad (42)$$

A Gaussian integration in the vector field $A_\mu(x)$ lead us to:

$$S_M^{(m \neq 0)} = \int d^D x \left[2(\partial^\mu B_{\mu\nu})^2 + \frac{J_\nu J^\nu}{2m^2} + \frac{2(\partial^\mu B_{\mu\nu})J^\nu}{m} + m^2 B_{\mu\nu} B^{\mu\nu} + B_{\mu\nu} T^{\mu\nu} \right] \quad (43)$$

The action above is the massive TEVR model coupled to source terms. To verify this result we redefine the fields $B_{\mu\nu}(x) \rightarrow \frac{\tilde{B}_{\mu\nu}(x)}{\sqrt{2}}$.

In order to show that the Maxwell-Proca theory can be related to the TEVR we use the expression (42) to perform a Gaussian integration in $B_{\mu\nu}(x)$:

$$S_M^{(m \neq 0)} = \int d^D x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{T_{\mu\nu} T^{\mu\nu}}{4m^2} + \frac{F^{\mu\nu} T_{\mu\nu}}{2m} - \frac{m^2}{2} A_\nu A^\nu + A_\nu J^\nu \right] \quad (44)$$

Where $F^{\mu\nu}(A) \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$.

The fact that the Maxwell-Proca and the TEVR model can be obtained from the same master action lead us to infer that they must have a common particle content. Both models describes spin 1 particles.

The dual map between those models can be obtained by considering their two point functions. In order to calculate them is necessary to use the functional generators:

$$Z_M^{(1)(m \neq 0)}[T_{\mu\nu}, J_\nu] = \int \mathcal{D}\tilde{B}_{\mu\nu}(x) e^{i \int d^D x \left[(\partial^\mu \tilde{B}_{\mu\nu})^2 + \frac{J_\nu J^\nu}{2m^2} + \frac{\sqrt{2}(\partial^\mu \tilde{B}_{\mu\nu})J^\nu}{m} + \frac{m^2}{2} \tilde{B}_{\mu\nu} \tilde{B}^{\mu\nu} + \frac{\tilde{B}_{\mu\nu} T^{\mu\nu}}{\sqrt{2}} \right]} \quad (45)$$

$$Z_M^{(2)(m \neq 0)}[T_{\mu\nu}, J_\nu] = \int \mathcal{D}A_\mu(x) e^{i \int d^D x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{T_{\mu\nu} T^{\mu\nu}}{4m^2} + \frac{F^{\mu\nu} T_{\mu\nu}}{2m} - \frac{m^2}{2} A_\nu A^\nu + A_\nu J^\nu \right]} \quad (46)$$

The first of the above functional generators is obtained from $S_M^{(m \neq 0)}$ through a Gaussian integration in the field $A_\mu(x)$ while the second one comes from an integration in the $B_{\mu\nu}(x)$ field. By performing functional derivatives we get the two point functions:

$$\frac{1}{2} \langle \tilde{B}_{\mu\nu}(x) \tilde{B}_{\alpha\beta}(y) \rangle = - \frac{\delta^2 Z_M^{(1)(m \neq 0)}[T_{\mu\nu}, J_\nu]}{Z_M^{(1)(m \neq 0)}[0, 0] \delta T_{\mu\nu}(x) \delta T_{\alpha\beta}(y)} \Big|_{T_{\alpha\beta}=J_\nu=0} \quad (47)$$

If we vary $Z_M^{(2)(m \neq 0)}[T_{\mu\nu}, J_\nu]$ with relation to the tensor sources, we should obtain a result that describes the same physics than the one obtained above since $Z_M^{(1)(m \neq 0)}[T_{\mu\nu}, J_\nu]$ as well $Z_M^{(2)(m \neq 0)}[T_{\mu\nu}, J_\nu]$ are originated by functional integrations of the same master action.

So, the two point function is given by the expression below:

$$-\frac{\delta^2 Z_M^{(2)(m \neq 0)}[T_{\mu\nu}, J_\nu]}{Z_M^{(2)(m \neq 0)}[0, 0] \delta T_{\mu\nu}(x) \delta T_{\alpha\beta}(y)} \Big|_{T_{\alpha\beta}=J_\nu=0} = \frac{1}{4m^2} \langle F^{\mu\nu}(x) F^{\alpha\beta}(y) \rangle + \frac{i}{2m^2} \delta^{(D-1)}(x-y) \quad (48)$$

From this last result we can obtain a dual map that relates the fields $B_{\mu\nu}(x)$ and $A_\mu(x)$ up to contact terms:

$$\tilde{B}_{\mu\nu} \longleftrightarrow \frac{\sqrt{2} F_{\mu\nu}(A)}{2m} \quad (49)$$

The inverse dual map can be obtained using the same argument employed in the above calculations. The difference lies in the fact that now we perform functional derivatives with respect to the vector sources. The two point function is given by:

$$\langle A_\mu(x) A_\nu(y) \rangle = -\frac{\delta^2 Z_M^{(2)(m \neq 0)}[T_{\mu\nu}, J_\nu]}{Z_M^{(1)(m \neq 0)}[0, 0] \delta J^\mu(x) \delta J^\nu(y)} \Big|_{T_{\alpha\beta}=J_\nu=0} \quad (50)$$

When we vary $Z_M^{(1)(m \neq 0)}$ with relation to the vector sources we should find a result that describes the same physics than the two point function calculated previously:

$$-\frac{\delta^2 Z_M^{(1)(m \neq 0)}[T_{\mu\nu}, J_\nu]}{Z_M^{(1)(m \neq 0)}[0, 0] \delta J^\mu(x) \delta J^\nu(y)} \Big|_{T_{\alpha\beta}=J_\nu=0} = -i \frac{\delta^{(D-1)}(x-y)}{m^2} + \frac{2}{m^2} \langle \partial^\alpha \tilde{B}_{\alpha\mu}(x) \partial^\gamma \tilde{B}_{\gamma\nu}(y) \rangle \quad (51)$$

This result allow us to obtain the inverse dual map that relates the fields $A_\mu(x)$ and $B_{\alpha\mu}(x)$ up to contact terms:

$$A_\mu(x) \longleftrightarrow \frac{\sqrt{2} \partial^\alpha \tilde{B}_{\alpha\mu}}{m} \quad (52)$$

The results obtained in this section are in agreement to the ones obtained in the Ref. 1 and represent an alternative way to relate the Maxwell-Proca model, which describes a spin 1 particle, to the massive TEVR model. This fact lead us to infer that it must describe a particle with this spin.

5.2 Massless phase and a real scalar field model

We can show that the massless phase of the TEVR model can be related to a spin 0 field by the use of the following master action:

$$S_M^{(m=0)} = \int d^D x \left[-C_\nu C^\nu + 2C^\nu (\partial^\mu B_{\mu\nu}) + C_\nu J^\nu + B_{\mu\nu} T^{\mu\nu} \right] \quad (53)$$

A Gaussian integration in the vector field lead us to the massless limit of the TEVR model coupled to sources:

$$S_M^{(m=0)} = \int d^D x \left[(\partial^\mu B_{\mu\nu})^2 + \frac{J_\nu J^\nu}{4} + \partial^\mu B_{\mu\nu} J^\nu + B_{\mu\nu} T^{\mu\nu} \right] \quad (54)$$

On the other hand, the master action (53) can be rewritten in an useful way by means of an integration by parts:

$$S_M^{(m=0)} = \int d^D x \left[-C_\nu C^\nu - F_c^{\mu\nu} B_{\mu\nu} + C_\nu J^\nu + B_{\mu\nu} T^{\mu\nu} \right] \quad (55)$$

Where $F_c^{\mu\nu}(x) \equiv \partial^\mu C^\nu(x) - \partial^\nu C^\mu(x)$.

The integration in the $B_{\mu\nu}(x)$ field is equivalent to integrating a functional delta function which imposes $F_c^{\mu\nu} = T^{\mu\nu}$. The solution is given below:

$$C_\nu = \partial_\nu \psi + C_\nu^T \quad (56)$$

The vector field decomposes in a longitudinal part, which is the solution of the homogeneous equation and in a transverse part which is given by:

$$C_\nu^T(x) = \frac{\partial^\mu T_{\mu\nu}}{\partial^2} \quad (57)$$

If we plug the above result in the action (55) we find:

$$S_M = \int d^D x \left[-\partial^\nu \psi \partial_\nu \psi + \partial_\nu \psi J^\nu - \left(\frac{\partial^\mu T_{\mu\nu}}{\partial^2} \right)^2 + \frac{\partial^\mu T_{\mu\nu} J^\nu}{\partial^2} \right] \quad (58)$$

The result obtained above shows that the massless phase of the TEVR model, up to source terms, is equivalent to the action of a real scalar field which describes a spin 0 particle. So, considering the results from the previous subsection we conclude that the master action technique can be used as another way for inferring the occurrence of the spin jumping.

The functional generators are given by:

$$Z_M^{(1)(m=0)}[T_{\mu\nu}, J_\nu] = \int dB_{\mu\nu}(x) e^{i \int d^D x \left[(\partial^\mu B_{\mu\nu})^2 + \frac{J_\nu J^\nu}{4} + \partial^\mu B_{\mu\nu} J^\nu + B_{\mu\nu} T^{\mu\nu} + \frac{\lambda}{2} G_{\lambda\mu\nu}(B) G^{\lambda\mu\nu}(B) \right]} \quad (59)$$

$$Z_M^{(2)(m=0)}[T_{\mu\nu}, J_\nu] = \int d\psi(x) e^{i \int d^D x \left[-\partial^\nu \psi \partial_\nu \psi + \partial_\nu \psi J^\nu - \left(\frac{\partial^\mu T_{\mu\nu}}{\partial^2} \right)^2 + \frac{\partial^\mu T_{\mu\nu} J^\nu}{\partial^2} \right]} \quad (60)$$

In the above expression we add a gauge fixing term because this action has the symmetry mentioned in (35).

A peculiarity that is present in the massless limit of the TEVR model is related to the obtainment of the dual map. Since there is no interaction terms that are linear in the tensor sources in $Z_M^{(2)(m=0)}$ we cannot find an invertible map by the same procedure adopted in the previous subsection. So, we restrict ourselves to infer that a scalar spin 0 field action and the massless TEVR are equivalent up to source terms. This result is again in accordance to our earlier results obtained by different approaches.

Although we cannot obtain an invertible dual map one can find, by the same procedure adopted in the previous section, a mapping between vector two point functions:

$$\langle \partial^\nu \psi(x) \partial^\mu \psi(y) \rangle = \langle \partial_\beta B^{\beta\nu}(x) \partial_\omega B^{\omega\mu}(y) \rangle = -i \frac{\delta^{(D-1)}(x-y)}{2} \quad (61)$$

The above result is clearly in accordance with the one given in the end of the Sec. 2 up to contact terms. Although this statement relates classical and quantum results it is not surprising since our calculations are being performed at tree level.

6 Concluding Remarks

This article consisted basically in verifying the spin jumping phenomena for the TEVR model under different approaches. Regarding the classical analysis, in the Sec. 2 we found this phenomena by means of the equations of motion while in the Sec. 3 we used the Hamiltonian formalism and found that its massless phase has reducible constraints as well as all the discontinuous models cited in this paper.

We could also understand this degree of freedom discontinuity under a quantum approach via unitarity analysis and master action technique presented in the Secs. 4 and 5, respectively.

As a future perspective we intend to present a phenomenological application of this degree of freedom discontinuity in the context of phase transitions with the mass as a parameter. To do so, we should derive its partition function and understand how this spin discontinuity appears in this context. So, regarding the massless phase, it is also necessary to have a clear picture of the role played by the constraint's reducibility in the construction of its gauge fixed Lagrangian.

7 Acknowledgements

We thank the referee for the interesting suggestions. This work was supported by CNPq (132619/2015-6).

8 Appendix A: A discontinuous vector model

Consider the spin 0 model described by the longitudinal excitations of a vector field:

$$\mathcal{L} = \frac{1}{2} \left[(\partial_\mu C^\mu)^2 + m^2 C_\mu C^\mu \right] \quad (62)$$

The equations of motion (E.O.M) are given by:

$$-\partial_\mu (\partial_\nu C^\nu) + m^2 C_\mu = 0 \quad (63)$$

The spin content of the theory can be obtained by contracting the above equation with a derivative:

$$(\square - m^2)(\partial_\mu C^\mu) = 0 \quad (64)$$

It is clear that we have massive scalar excitations described by $\partial_\mu C^\mu$. On the other hand the massless model presents a local freedom, which is analogous to the massless TEVR one:

$$\delta C_\mu(x) = \partial^\nu \Gamma_{[\nu\mu]}(x) \quad (65)$$

Where $\Gamma_{[\nu\mu]}(x)$ is an arbitrary antisymmetric field. The local transformation above is reducible since it is invariant by $\delta \Gamma_{[\nu\mu]}(x) \rightarrow \bar{\Gamma}_{[\nu\mu]}^T(x)$ where T designates transversality. The E.O.M are:

$$\partial_\mu (\partial_\nu C^\nu) = 0 \quad (66)$$

This equation determines that $\partial_\nu C^\nu$ is a constant. So, requiring that the fields falls to zero at spacetime infinity force us to consider the trivial solution. The model becomes trivial in the massless limit. It can be understood by the vanishing of its interparticle potential in this limit (Ref.8).

9 Appendix B: Spin projectors

Using the spin-0 and spin-1 projection operators acting on vector fields, respectively,

$$\omega_{\mu\nu} = \frac{\partial_\mu \partial_\nu}{\square}, \quad \theta_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\square}, \quad (67)$$

as building blocks, one can define the projection operators in D dimensions acting on antisymmetric rank-2 tensors. First, we define the transversal and longitudinal operators as follows

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\square} \quad \omega_{\mu\nu} = \frac{\partial_\mu \partial_\nu}{\square}. \quad (68)$$

The above set of operator satisfies

$$\theta^2 = \theta, \quad \omega^2 = \omega \quad \text{and} \quad \theta\omega = \omega\theta = 0. \quad (69)$$

On the other hand, the set of the antisymmetric four-dimensional Barnes-Rivers operator are given by

$$P_{\mu\nu,\alpha\lambda}^{[1b]} = \frac{1}{2}(\theta_{\mu\alpha}\theta_{\nu\lambda} - \theta_{\mu\lambda}\theta_{\nu\alpha}), \quad (70)$$

$$P_{\mu\nu,\alpha\lambda}^{[1e]} = \frac{1}{2}(\theta_{\mu\alpha}\omega_{\nu\lambda} + \theta_{\nu\lambda}\omega_{\mu\alpha} - \theta_{\mu\lambda}\omega_{\nu\alpha} - \theta_{\nu\alpha}\omega_{\mu\lambda}). \quad (71)$$

They satisfy the very simple algebra

$$\begin{aligned} (P^{[1b]})^2 &= P^{[1b]}, \quad (P^{[1e]})^2 = P^{[1e]}, \\ P^{[1b]}P^{[1e]} &= P^{[1e]}P^{[1b]} = 0. \end{aligned} \quad (72)$$

10 Appendix C: The Dirac brackets of the massive TEVR model

The massive phase of the TEVR model has just second class constraints and it is related to the fact that the Lagrange multipliers are determined. So, according to the Dirac-Bergmann procedure we need to consider a reduced phase space given by the Dirac brackets. First of all, its necessary to build the constraint matrix:

$$\Gamma_{mn}^{ij}(x, y) = \begin{pmatrix} \{\Omega^{ij}(x), \Omega_{mn}(y)\} & \{\Omega^{ij}(x), \alpha_{mn}(y)\} \\ \{\alpha^{ij}(x), \Omega_{mn}(y)\} & \{\alpha^{ij}(x), \alpha_{mn}(y)\} \end{pmatrix} = m^2 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \frac{(\delta_m^i \delta_n^j - \delta_m^j \delta_n^i)}{2} \delta^{(D-1)}(x - y) \quad (73)$$

The inverse of $\Gamma_{mn}^{ij}(x, y)$ is given by:

$$\Gamma_{mn}^{ij}(x, y)^{-1} = \frac{1}{m^2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \frac{(\delta_m^i \delta_n^j - \delta_m^j \delta_n^i)}{2} \delta^{(D-1)}(x - y) \quad (74)$$

Now we can obtain the Dirac brackets that generates the reduced phase space of the system:

$$\{A(x), B(y)\}_D = \{A(x), B(y)\} - \int d^{D-1}u d^{D-1}v \{A(x), \chi_{ij}^a(u)\} \Gamma_{mn}^{ij}(ab)(u, v)^{-1} \{\chi^{b\ mn}(v), B(y)\} \quad (75)$$

Where $\chi_{ij}^a(u) = \begin{pmatrix} \Omega_{ij}(u) \\ \alpha_{ij}(u) \end{pmatrix}$. The reduced phase space is given by the nonvanishing Dirac brackets:

$$\{B^{0i}(x), \pi_{0j}(y)\}_D = \frac{\delta_j^i}{2} \delta^{(D-1)}(x - y) \quad (76)$$

$$\{B^{ij}(x), B_{0l}(y)\}_D = \frac{\partial^{[j} \delta^{i]}_l}{2m^2} \delta^{(D-1)}(x - y) \quad (77)$$

Where $[,]$ denotes antisymmetrization.

References

- [1] - A. Smailagic and E. Spallucci, Phys. Rev. D 61 067701 (2000).
- [2] - D. Dalmazi, R.C. Santos, Phys. Rev. D 84 045027 (2011).
- [3] E. Harikumar and M. Sivakumar, Nucl.Phys. B565 385-396 (2000).
- [4] M. Mondragon and M. Montesinos, J. Math. Phys. 47, 022301 (2006).
- [5] - K. Kampf, J. Novotn and J. Trnka, Eur. Phys. J. C 50 385 (2007).
- [6] -K. Kampf, J. Novotn and J. Trnka, Fizika B 17 349 (2008).
- [7] - K. Kampf, J. Novotn and J. Trnka, Phys. Rev. D 81 116004 (2010).
- [8] G. B. de Gracia and G. P. de Brito, Int. J. Mod. Phys. A 31 1650070 (2016).
- [9] - M. Kalb and D. Ramond, Phys. Rev. D 9 2273 (1974).
- [10] - A. Smailagic and E. Spallucci, J. Phys. A 34 L 435 (2001).
- [11] - E. Di Grezia and S. Esposito, Int. Theor. Phys. 43 445 (2004).
- [12] - P.A.M. Dirac, *Lectures on Quantum Mechanics* (Belfer Graduate school of Science, Yeshiva University, NY 1964).
- [13] - M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* Princeton University Press, (1992).
- [14] - D.G. Boulware and S. Deser, Phys. Rev. D 6: 3368 (1972).
- [15] - J. Schwinger, *Particles, Sources and Fields*, Perseus Books Publishing, Vol. I (1998).
- [16] - S. Deser and R. Jackiw, Phys. Rev. Lett. B 139 371 (1984).
- [17] - S. Deser, R. Jackiw and S. Templeton, Annals Phys. 140 372 (1984).
- [18] - I.A. Batalin and E.S. Fradkin, Phys. Lett. 122B 157 (1983).